### Theorem: Associative Laws for Sets

Let L, M, and N be sets. Then the following two rules apply:LL MM NN == LL MM NN

Proof:

First, we prove that L ∪ (M ∪ N) = (L ∪ M ) ∪ N. For this purpose we follow the defini-

tion of equality of sets: If we can show that the set on the left side is a subset of the set on the right side and vice versa, then it follows that the two sets are equal. This is exactly how

We first show that L ∪ M) ∪ N L ∪ (M ∪ N)L ∪ (M ∪ N) is ⊆ (L ∪ M) ∪ Nx (L ∪ M) ∪ N. For this we must show that eachL M ∪ Nx ∈xN Lx ∈ Nx x ∈ L ∪x ∈x ∈ we want to proceed:

this set, so x ∈ L ∪ (M ∪ N). Since lies in the union of and , must lie in at element from is also located in . So let be any element of

is , then

x ∈ (L ∪ M) ∪ N. If , then

or in or in both. If

(this is left to you for

practice). This leads to the first part of the claim, namely L ∪ (M ∪ N) = (L ∪ M) ∪ N.

Let ∩ M) ∩ NN □x ∈ L ∩ (M ∩ N)x M . Then N x L ∩ (M ∩ N) ⊇ (L ∩ M) ∩ NL ∩ (M ∩ N) = (L ∩ M) ∩ The second part of the assertion is shown in a similar way:

that is in both and . It follows further that

. Thus it follows that

Analogously, one can show that (this is left to you for practice). This leads to the second part of the claim, namely

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